Discrete Image Hashing Using Large Weakly Annotated Photo Collections

Hanwang Zhang†, **Na Zhao**†, **Xindi Shang**†, **Huanbo Luan**‡, **Tat-seng Chua**†

† National University of Singapore

‡ Tsinghua University

†hanwang,zhaona,shangxin,chuats@comp.nus.edu.sg, ‡luanhuanbo@gmail.com

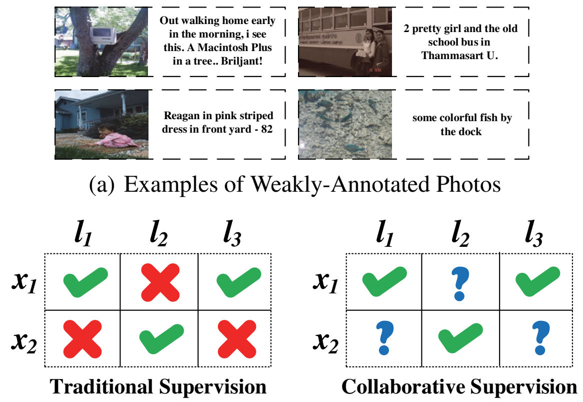
Abstract

We address the problem of image hashing by learning binary codes from large and weakly supervised photo collections. Due to the explosive growth of user-generated media on the Web, this problem is becoming critical for large-scale visual applications like image retrieval. While most existing hashing methods fail to address this challenge well, our method shows promising improvement due to the following two key advantages. First, we formulate a novel hashing objective that can effectively mine implicit weak supervision by collaborative filtering. Second, we propose a discrete hashing algorithm, offered with efficient optimization, to overcome the inferior optimizations in obtaining binary codes from real-valued solutions. In this way, our method can be considered as a weakly-supervised discrete hashing framework which jointly learns image semantics and their corresponding binary codes. Through training on one million weakly annotated images, our experimental results demonstrate that image retrieval using the proposed hashing method outperforms the other state-of-the-art ones on image and video benchmarks.

Introduction

By encoding data into short binary codes, hashing supports efficient storage of huge amount of data in memory and make fast similarity search possible. For example, similarity calculation of a query in the database will be reduced to efficient bit operations, and subsequent ranking in Hamming space can be performed in (sub-) linear (by Hamming ranking) or even constant time (by lookup table). Therefore, hashing technique is not only indispensable in processing today's multimedia content on the Web, where people usually require to find results from billions of images and videos within one second (Jegou, Douze, and Schmid 2011; Wang, Kumar, and Chang 2012), but also in fundamental machine learning scenarios where fast and scalable kernel or nearest neighbor constructions are needed (Kulis and Grauman 2009; Liu et al. 2012).

Existing hashing methods fall into unsupervised and (semi-) supervised categories according to whether labeled data are required. Unsupervised hashing seeks hash functions that preserve data similarity in the original feature space. For example, Locality Sensitive Hashing (LSH) designs random projections that map features to binary codes so that similar data will have same codes with high probability (Gionis et al. 1999; Kulis and Grauman 2009); while Spectral Hashing (SH) learns hash functions that preserves the graph structure of the original feature space (Weiss, Torralba, and Fergus 2009; Kong and Li 2012; Liu et al. 2011). However, unsupervised hashing does not guarantee the semantic similarity between data, especially for visual data. This is due to the well-known *semantic gap* where the high-level semantics of visual content often differs from low-level visual features (Smeulders et al. 2000). Therefore, (semi-) supervised hashing that exploits semantic label information is shown to be effective. Popular methods are Min-Loss Hashing (Norouzi and Blei 2011), CCA-Hashing (Gong et al. 2013) and Semi-Supervised Hashing (Wang, Kumar, and Chang 2012). They generally learn hash functions that mini-


Out walking home early in the morning, i see this. A Macintosh Plus in a tree.. Briljant!
2 pretty girl and the old school bus in Thammasart U.
Reagan in pink striped dress in front yard - 82
some colorful fish by the dock

(a) Examples of Weakly-Annotated Photos

Traditional Supervision

	l_1	l_2	l_3
x_1	✓	✗	✓
x_2	✗	✓	✗

Collaborative Supervision

	l_1	l_2	l_3
x_1	✓	?	✓
x_2	?	✓	?

(b) Traditional vs. Collaborative Supervision

Figure 1: (a) User-generated annotations for images are usually noisy and incomplete. These examples are from SBU 1M dataset (Ordonez, Kulkarni, and Berg 2011), which is used as the training data in this paper. (b) The key difference between traditional and collaborative supervision is that the latter does not consider unobserved labels as negative. This is crucial for training on weakly-labeled dataset.

mize pair-wise similar/dissimilar Hamming distance cost according to labeling supervision. Supervised hashing requires sufficiently large training data to achieve good generalizations for new samples. However, obtaining high-quality labeled data is usually prohibitively expensive in practice.

In recent years, with the surge in popularity of social networks, users have generated almost inexhaustible multimedia contents with weak annotations such as Flickr (see Figure 1(a)). Therefore, it is interesting to investigate whether such large amount of weakly labeled collection can help in learning image hash codes. In particular, we aim to tackle the following challenges that few existing supervised hashing techniques address:

- **Quantization Loss**. The discrete constraints imposed on binary code learning lead to NP-hard mixed-integer programming (Håstad 2001). Therefore, most supervised hashing methods resort to relaxing the problem by discarding the discrete constraints, *e.g.*, solving continuous problem and then followed by thresholding or minimizing quantization loss (Gong et al. 2013). Although relaxation makes the original problem feasible, its resulting hash functions are less effective due to the accumulated quantization error. In real large-scale applications which require longer code length for sufficient precision, the performance drop caused by quantization loss will be more severe.
- **Multi- vs. Single-Label**. Training images collected from social networks are multi-labeled in nature. However, most supervised hashing methods focus on two-class pairwise relations; for multi-labeled data, the relation between two images is no longer the simple similar/dissimilar relations, but the more complex multi-level semantic similarities. Though some works (Zhao et al. 2015; Norouzi, Blei, and Salakhutdinov 2012) can transform the multi-level similarities into ranking objectives, they do not scale up to large-scale training data since they require combinatorial number of ranking pairs or triplets for training.
- **Weak vs. Full Supervision**. As compared to full supervision, where data are fully labeled across all class labels, weakly labeled data are generally not annotated with complete class labels. This is reasonable since casual users are reluctant to provide complete tags. As shown in Figure 1(a), as compared to the entire English vocabulary, annotations are generally sparse and weak. Unfortunately, traditional supervised hashing methods strictly enforce the missing labels as negative, and hence inevitably harm the subsequent semantic understanding.

In this paper, by addressing the above challenges, we propose a novel hashing framework for learning hash functions by using a large and weakly labeled photo collection. We propose to bring Collaborative Filtering (CF), which has been successfully applied in discovering the weak relationship between many users and items (Koren, Bell, and Volinsky 2009), to analyze the weak but abundant associations between images and labels and then predict the new (unobserved) image-label associations (see Figure 1(b)). The key motivation is that CF can elegantly avoid modeling the high portion of missing annotations by efficient sparse matrix factorizations, and CF naturally supports multi-labeled training images. We develop a formulation that alternatively optimizes the binary codes that involve CF and quantization loss. For solution, we propose a discrete optimization method which directly learns the binary codes and the corresponding hash functions. Thus, the quantization loss is theoretically guaranteed to be minimized without any relaxation. Our algorithm only requires simple matrix multiplications and inversion of small matrices, and thus can be efficiently applied in large-scale settings. To demonstrate the effectiveness of the proposed method, we train our model on a lM Flickr photo collection with over 30K number of weak labels and test its performance in retrieval on two challenging image and video benchmarks. Experimental results show promising improvement over other state-of-the-art hashing methods.

Related Work

For space limitation, we only review recent hashing methods that cover the idea of discrete optimization and multi-label supervision. For comprehensive reviews, please refer to (Wang et al. 2014; Grauman and Fergus 2013).

The most widely used discrete hashing technique is perhaps Iterative Quantization (ITQ) (Gong et al. 2013). It minimizes the quantization loss by alternatively learning the binary codes and the hashing function. However, it has two drawbacks. First, ITQ requires the hashing function be orthogonal projection, which is not applicable in many other supervised objectives. Second, its optimization is a post-processing after learning the major objective (*e.g.*, geometry or semantic preserving similarities). Thus, ITQ is sub-optimal to the objective task. Discrete Graph Hashing (Liu et al. 2014a) and Supervised Discrete Hashing (Shen et al. 2015) are recently proposed methods based on discrete optimization. They demonstrate that jointly minimizing quantization loss and major objective function through discrete optimization can improve the hashing performance. Our work is also an advocate of discrete optimization but focuses on the problem of weakly supervised learning with multi-label data. Weakly Supervised Hashing (Mu, Shen, and Yan 2010) focuses on hashing trained by partially labeled training data, where the definition of “weak” is more similar to “semi-supervised” while we focus on “incomplete” labeling. Besides, it needs kernel and CCCP (Concave-Convex Procedure) optimization which does not scale up to large-scale data and labels. (Gong et al. 2013) tackle multi-labeled data as cross-modality fusion and apply CCA to obtain the correlated representation between the data and label. Then, the representations quantize to be hashing codes by ITQ. (Zhao et al. 2015) cast multi-label supervision into ranking supervision according to the number of shared labels. However, these two methods cannot be applied in our weakly labeled case since the noisy incomplete labels of data will mislead the training. The idea of using collaborative filtering to handle weakly labeled data is similar to (Liu et al. 2014b). However, their work neglects to handle the incompleteness of labels, which is the most essential motivation of collaborative filtering. Based on the above discussion, to our best knowledge, this work is the first to learn hash functions for large-scale, weakly labeled data with explicit discrete optimization. Thanks to the large-scale learning, the learned hash functions can well generalize to test data even if they are very different from training. This allows us to, say, train on images but test on videos.

Problem Formulation

We use bold upper-case letter $\mathbf{A}$ as matrix, bold lower-case letter with subscript $\mathbf{a}_i$ as the i-th column of $\mathbf{A}$, and upper-case letter with subscript A_{ij} as the entry at i-th row and j-th column. We have n training images $\{\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n\}$, where $\mathbf{x}_i \in \mathbb{R}^d$ represents the feature vector in d-dimensional space, and a column of data matrix $\mathbf{X} \in \mathbb{R}^{d\times n}$. Our goal is to generate b hash codes $\mathbf{B} \in \{\pm 1\}^{b\times n}$ for the images. The advantage of denoting the binary codes by $\{\pm 1\}$ is that the Hamming distance between image i and j is a strict monotonically decreasing function of the vector product $\mathbf{b}_i^T \mathbf{b}_j$ and simplifies the formulation of learning to hash. In this paper, we only consider linear functions,

$$\mathbf{b}_i = \mathrm{sgn}(\mathbf{W}^T \mathbf{x}_i), \tag{1}$$

where $\mathbf{W} \in \mathbb{R}^{d\times b}$ is the matrix of b linear hash functions and $\mathrm{sgn}(\cdot)$ is the element-wise function that $\mathrm{sgn}(x) = 1$ if $x > 0$ and -1 otherwise. We learn $\mathbf{W}$ from the labeled data. Each image has an m-dimensional annotation (*i.e.*, m words), $\mathbf{a}_i \in \mathbb{R}^m$, which is a column of label matrix $\mathbf{A} \in \mathbf{R}^{m\times n}$. Note that the labels are weakly supervised, that is, most entries of $\mathbf{A}$ is empty. In particular, we use $A_{ij} = 1$ to denote that image j is labeled with word i and $A_{ij} = 0$ if we have no observation on whether image j is labeled with i. Note that it is the crucial difference from traditional multi-label supervision which considers $A_{ij} = 0$ as negative.

Collaborative Supervision and Quantization Loss

We propose to use Collaborative Filtering (CF) to effectively uncover the semantic relations hidden in weak supervision. In particular, we focus on the matrix factorization-based CF, which has been demonstrated to be one of the most successful CF methods (Koren, Bell, and Volinsky 2009). Suppose $\mathbf{u}_i \in \mathbb{R}^b$ and $\mathbf{c}_j \in \mathbb{R}^b$ are the latent vectors of label i and image j, we expect that inner product $\mathbf{u}_i^T \mathbf{c}_j$ is large if image j is annotated with label i, and small otherwise. Although Collaborative Hashing (Liu et al. 2014b) is also based on this idea, we argue that counting missing labels caused by weak supervision as negative labels, *i.e.*, $\mathbf{u}_i^T \mathbf{c}_j$ should be small if $A_{ij} = 0$, is not reasonable. Therefore, our collaborative supervision model strictly stick to the definition of classic CF (Koren, Bell, and Volinsky 2009), by only learning from the observed annotations (*i.e.*, $A_{ij} \neq 0$),

$$\min_{\mathbf{U},\mathbf{C}} \left\| \left(\mathbf{A} - \mathbf{U}^T \mathbf{C}\right) \odot \mathbf{A} \right\|_F^2 + \lambda_1 R(\mathbf{U}) + \lambda_2 R(\mathbf{C}), \tag{2}$$

where $\odot$ is the Hadamard (element-wise) matrix multiplication, which does not count the loss if $A_{ij} = 0$; $R(\cdot)$ is any regularization that scales the resultant $\mathbf{U}$ and $\mathbf{C}$.

Given code $\mathbf{b}_i = \mathrm{sgn}(\mathbf{c}_i) = \mathrm{sgn}(\mathbf{W}^T \mathbf{x}_i)$, it is easy to show that the smaller the quantization loss $\|\mathbf{b}_i - \mathbf{c}_i\|$ is, the better the resulting binary codes will preserve the desired solution in Eq. (2). Therefore, we expect that the (linear) hash functions can minimize the quantization error:

$$\min_{\mathbf{W}} \|\mathbf{B} - \mathbf{W}^T \mathbf{X}\|_F^2 + \lambda \|\mathbf{W}\|_F^2,\ s.t.\ \mathbf{B} = \mathrm{sgn}(\mathbf{W}^T \mathbf{X}). \tag{3}$$

where $\lambda\|\mathbf{W}\|_F^2$ scales the linear model $\mathbf{W}$ and hence scales $\mathbf{C}$. However, it is impractical to directly solve $\mathbf{W}$ since $\mathrm{sgn}(\cdot)$ is non-differentiable. As in ITQ (Gong et al. 2013), we cast the problem into an iterative procedure:

$$\min_{\mathbf{W},\mathbf{B}} \|\mathbf{B} - \mathbf{W}^T \mathbf{X}\|_F^2 + \lambda \|\mathbf{W}\|_F^2,\ s.t.\ \mathbf{B} \in \{\pm 1\}^{b\times n}. \tag{4}$$

As compared to the two-step methods such as ITQ which is sub-optimal, we propose to jointly optimize the collaborative supervision objective in Eq. (2) and the quantization loss in Eq. (4). The overall objective function is formulated as:

$$\begin{aligned} &\min_{\mathbf{U},\mathbf{B},\mathbf{W}} \underbrace{\left\| \left(\mathbf{A} - \mathbf{U}^T \mathbf{B}\right) \odot \mathbf{A} \right\|_F^2}_{\text{Collaborative Supervision}} + \underbrace{\lambda_1 \|\mathbf{B} - \mathbf{W}^T \mathbf{X}\|_F^2}_{\text{Quantization Loss}} + \lambda_2 \|\mathbf{W}\|_F^2, \\ &s.t.\ \mathbf{B} \in \{\pm 1\}^{b\times n},\ \mathbf{U}\mathbf{U}^T = \mathbf{I}. \end{aligned} \tag{5}$$

Note that we instantiate the regularization for $\mathbf{U}$ as the orthogonal constraint, which scales $\mathbf{U}$ and decorrelate the basis latent features of the labels as well. This objective function is not convex, we will design an iterative algorithm to find a local optimum.

Optimization

The proposed algorithm tackles three challenges in optimizing Eq. (5): 1) the Hadamard multiplication with $\mathbf{A}$; 2) the orthogonal constraint $\mathbf{U}\mathbf{U}^T = \mathbf{I}$; and 3) the discrete constraint $\mathbf{B} \in \{\pm 1\}^{b\times n}$. We will detail our solutions in the following three subproblems.

Fix B and U, update W This subproblem reduces to a simple linear regression: $\min_{\mathbf{W}} \lambda_1 \|\mathbf{B} - \mathbf{W}^T \mathbf{X}\|_F^2 + \lambda_2 \|\mathbf{W}\|_F^2$, which can be solved efficiently by

$$\mathbf{W} \leftarrow \left(\mathbf{X}\mathbf{X}^T + \frac{\lambda_2}{\lambda_1}\mathbf{I}\right)^{-1} \mathbf{X}\mathbf{B}^T. \tag{6}$$

Since $(\mathbf{X}\mathbf{X}^T + \lambda_2/\lambda_1 \mathbf{I})^{-1}$ is fixed during optimization, we can precompute and store its inversion for efficiency.

Fix B and W, update U By expanding the quadratic terms according to $\mathbf{U}$ in Eq. (5), the subproblem of updating $\mathbf{U}$ can be rewritten as:

$$\begin{aligned} &\min_{\mathbf{U}\mathbf{U}^T = \mathbf{I}} F(\mathbf{U}) = \\ &-\sum_i \left(\sum_{j\in\mathcal{I}_i} A_{ij} (\mathbf{c}_j^T + \mathbf{b}_j^T) \right) \mathbf{u}_i + \tfrac{1}{2} \sum_i \sum_{j\in\mathcal{I}_i} \mathbf{u}_i^T \mathbf{b}_j \mathbf{b}_j^T \mathbf{u}_i \\ &= -tr(\mathbf{A}\mathbf{B}^T\mathbf{U}) + \tfrac{1}{2} \sum_i \mathbf{u}_i^T \widetilde{\mathbf{B}}_i \mathbf{u}_i, \end{aligned} \tag{7}$$

where $\mathcal{I}_i$ denotes the nonzero indices in the i-th row of $\mathbf{A}$, and $\widetilde{\mathbf{B}}_i = \sum_{j\in\mathcal{I}_i} \mathbf{b}_j \mathbf{b}_j^T$. We apply a gradient descent with orthogonality constraint (Wen and Yin 2013) to solve the challenging quadratic programming in Eq. (7). Suppose the learning rate is η, the update rule at each iteration is:

$$\mathbf{U}^T \leftarrow \mathbf{U}^T - \eta\mathbf{R}\left(\mathbf{I} + \frac{\eta}{2}\mathbf{L}^T\mathbf{R}\right)^{-1}\mathbf{L}^T\mathbf{U}^T, \tag{8}$$

where $\mathbf{R} = [\mathbf{G}^T, \mathbf{U}^T]$, $\mathbf{L} = [\mathbf{U}^T, -\mathbf{G}^T]$, and $\mathbf{G}$ is the gradient of $F(\mathbf{U})$: $\mathbf{G} = -\mathbf{B}\mathbf{A}^T + [\widetilde{\mathbf{B}}_1\mathbf{u}_1, ..., \widetilde{\mathbf{B}}_m\mathbf{u}_m]$. In practice, before we start the update iteration, we apply Gram-Schmidt process to force $\mathbf{U}\mathbf{U}^T = \mathbf{I}$ since the orthogonality constraint may deteriorate after a certain number of iterations due to numeric instability.

Fix $\mathbf{U}$ and $\mathbf{W}$, update $\mathbf{B}$ This binary constraint in this subproblem is generally NP-hard (Håstad 2001). Therefore, many methods solve a relaxed problem by dropping the discrete constraint but obtain a sub-optimal result. Here, we propose an efficient algorithm that enforces the constraint to directly achieve discrete $\mathbf{B}$.

Due to the Hadamard product of $\mathbf{A}$ in Eq. (5), we cannot derive a solution for $\mathbf{B}$ in matrix formulation. Fortunately, since column $\mathbf{b}_j$ independently contributes to the loss, we can update $\mathbf{b}_j$ in parallel. Noting the fact that $\mathbf{U}\mathbf{U}^T = \mathbf{I}$ and $\mathbf{b}_j^T\mathbf{b}_j = b$, the expansion of Eq. (5) according to the j-th column of $\mathbf{B}$ is:

$$\begin{aligned} &const - 2\left(\sum_{i\in\mathcal{I}_j} A_{ij}\mathbf{u}_i^T\right)\mathbf{b}_j + \underbrace{\mathbf{b}_j^T\mathbf{U}\mathbf{U}^T\mathbf{b}_j + \mathbf{b}_j^T\mathbf{b}_j}_{\text{constant}} \\ &-\mathbf{b}_j^T\left(\sum_{i\notin\mathcal{I}_j}\mathbf{u}_i\mathbf{u}_i^T\right)\mathbf{b}_j - 2\lambda_1\mathbf{x}_j^T\mathbf{W}\mathbf{b}_j, \end{aligned} \tag{9}$$

where $\mathcal{I}_j$ is the nonzero indices of the j-th column of $\mathbf{A}$. Thus, this $\mathbf{B}$-subproblem reduces to *maximizing* the following n independent problems:

$$\max_{\mathbf{b}_j\in\{\pm1\}^b} \frac{1}{2}\mathbf{b}_j^T\mathbf{H}\mathbf{b}_j + \mathbf{p}^T\mathbf{b}_j, \tag{10}$$

where $\mathbf{H} = \sum_{i\notin\mathcal{I}_j}\mathbf{u}_i\mathbf{u}_i^T$, $\mathbf{p}^T = \lambda_1\mathbf{x}_j^T\mathbf{W} + \sum_{i\in\mathcal{I}_j} A_{ij}\mathbf{u}_i^T$.

We maximize the function $f(\mathbf{b}) = \frac{1}{2}\mathbf{b}^T\mathbf{H}\mathbf{b} + \mathbf{p}^T\mathbf{b}$ in Eq. (10) iteratively. In particular, at the $(t+1)$-th iteration, we maximize a surrogate function $\tilde{f}_t(\mathbf{b}) = f(\mathbf{b}^{(t)}) + \nabla f^T(\mathbf{b}^{(t)})(\mathbf{b} - \mathbf{b}^{(t)})$, where we denote the maximum solution as $\mathbf{b}^{(t+1)}$. Since $f(\mathbf{b})$ is a convex function ($\mathbf{H}$ is semi-positive definite), we have the fact $f(\mathbf{b}^{(t+1)}) \geq \tilde{f}_t(\mathbf{b}^{(t+1)}) \geq \tilde{f}_t(\mathbf{b}^{(t)}) = f(\mathbf{b}^{(t)})$. Therefore, we can ensure that the sequential solution $\{\mathbf{b}^{(t)}\}$ will iteratively converge to a local maximum solution to Eq. (10). According to the above analysis, we can easily derive the update rule for $\mathbf{b}_j$ as:

$$\mathbf{b}_j \leftarrow \text{sgn}(I(\nabla f(\mathbf{b}_j), \mathbf{b}_j)) = \text{sgn}\left(I(\mathbf{H}\mathbf{b}_j + \mathbf{p}, \mathbf{b}_j\right), \tag{11}$$

where $I(x, y)$ is an element-wise function such that $I(x, y) = x$ if $x \neq 0$ and $I(x, y) = y$ otherwise. The above update rule states that the updated $\mathbf{b}_j$ should have the same signs as $\nabla f(\mathbf{b}_j)$; if the derivative is zero at certain entries, we do not update the corresponding bits.

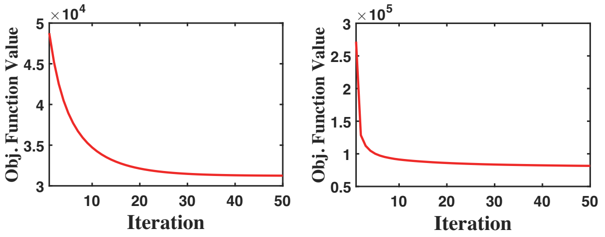
Algorithm 1: Discrete Weakly-Supervised Hashing

```
Input  : $\mathbf{X} \in \mathbb{R}^{d\times n}$: training image features,
         $\mathbf{A} \in \{0,1\}^{m\times n}$: weakly-labeled annotation,
         $b$: bit size,
         $\eta$: learning rate,
         $\lambda_1$ and $\lambda_2$: trade-off parameter
Output: $\mathbf{W} \in \mathbb{R}^{d\times b}$: linear hashing model
1  Initialization:
   $(\mathbf{U}^{(0)}, \mathbf{B}^*) = \arg\min_{\mathbf{U}\mathbf{U}^T=\mathbf{B}\mathbf{B}^T=\mathbf{I}} \|\mathbf{A} - \mathbf{U}^T\mathbf{B}\|_F^2$,
   $\mathbf{B}^{(0)} = \text{sgn}(\mathbf{B}^*)$, $\mathbf{W}^{(0)}$ is random, $t = 0$
2  repeat
3      W-subproblem:
       $\mathbf{W}^{(t+1)} \leftarrow \left(\mathbf{X}\mathbf{X}^T + \lambda_2/\lambda_1\mathbf{I}\right)^{-1}\mathbf{X}\mathbf{B}^{(t)^T}$;
4      U-subproblem:
       $\mathbf{U}^{(t+1)^T} \leftarrow \mathbf{U}^{(t)^T} - \eta\mathbf{R}\left(\mathbf{I} + \frac{\eta}{2}\mathbf{L}^T\mathbf{R}\right)^{-1}\mathbf{L}^T\mathbf{U}^{(t)^T}$
       according to Eq. (8);
5      B-subproblem (parallel updating):
       for j=1 to n do
6          $\mathbf{b}_j^{(t+1)} \leftarrow \text{sgn}\left(I(\mathbf{H}\mathbf{b}_j^{(t)} + \mathbf{p}, \mathbf{b}_j^{(t)}\right)$ according to
           Eq. (11);
7      end
8      $t \leftarrow t + 1$;
9  until converge;
10 return $\mathbf{W}^{(t)}$
```

Algorithmic Analysis

Initialization Since the proposed formulation in Eq. (6) is non-convex, the above optimization needs a good choice of initial solution. Thus, we here suggest a strategy to obtain one. For initializing $\mathbf{U}$ and $\mathbf{B}$, we solve an relaxed $\min_{\mathbf{U}\mathbf{U}^T=\mathbf{B}\mathbf{B}^T=\mathbf{I}} \|\mathbf{A} - \mathbf{U}^T\mathbf{B}\|_F^2$ without specially dealing with the unobserved labels. This problem can be efficiently solved with SVD (Liu et al. 2014b) if we initialize $\mathbf{B}$ or $\mathbf{U}$ by Gram-Schmidt process. Then, the initialization of $\mathbf{B}$ is $\mathbf{B} \leftarrow \text{sgn}(\mathbf{B})$. The underlying heuristic of this initialization is that the quantization loss minimization is meaningless unless we have already obtained a good real-valued solution of the major objective.

Complexity In training stage, the major space consumption is the image feature matrix $\mathbf{X}$ which requires $\mathcal{O}(dn)$. Fortunately, state-of-the-art image features are usually sparse (Donahue et al. 2013). In our case, it requires only 3-GB memory to store 1M images. Other space consumptions including $\mathbf{U}$, $\mathbf{B}$, and the precomputed $\mathbf{X}\mathbf{X}^T$ in Eq. (6) and $\mathbf{G}$ in Eq. (8) are $\mathbf{O}(bn + bm + bd)$, which is moderate since d and b are small. At each training iteration, the time consumption for W-subproblem in Line 3 Alg (1) is only $\mathcal{O}(d^2bn)$ since the inversion can be computed off-line. For U-subproblem in Line 4, it requires $\mathcal{O}(b^3 + mb^2)$ for $b \times b$-size eigen-decomposition and matrix multiplications. For B-subproblem in Line 5, it requires $\mathcal{O}(sb^2 + db)$ for constructing $\mathbf{H}$ and $\mathbf{p}$, where $s = \mathcal{O}(n)$ is the number of nonzero entries in $\mathbf{A}$. For the update rule, it requires only $O(b^2)$. By parallel updating, it requires $\mathcal{O}(\frac{n}{p}(sb^2 + db))$ in total, where p is number of parallel computing threads. Based on the above analysis, we can see that the time complexity of



(a) With Initialization (b) W/O Initialization

Figure 2: Convergence curve of the objective function using a sub-sampled dataset with 0.1M data, $b = 32$, $\lambda_1 = 1$, $\lambda_2 = 1e - 4$.

our algorithm is linear to the sample size: $\mathcal{O}(Tn)$, where T is the number of iterations. In our experiments, we found that we usually needed $T \leq 50$ for convergence. On our Intel i7 6-core machine with 3.0Ghz CPU and 64-GB memory, we needed about 1 hour per iteration for training 1M data. In testing stage, we only needed to store $\mathbf{W}$ of the size $d \times b$ to generate hash codes for images.

Convergence Due to space limit, we omit rigorous proof for convergence. Generally, one can easily complete the proof based on the fact that the proposed algorithm monotonically decreases towards the lower-bounded objective function. We show the convergence of the algorithm in Figure 2, which implies that proper initialization is necessary.

Experiments

Dataset

For training the hashing function, we used a large weakly-annotated image dataset called **SBU** (Ordonez, Kulkarni, and Berg 2011). It contains 1M images with user-generated captions. For each image, the words of caption can be considered as the observed labels. After removing stop- and low-frequency words, we had 30,456 labels in total, *i.e.*, matrix $\mathbf{A}$ is of the size $30,456 \times 1M$. In order to simulate practical search scenario, where application database is usually different from training, we used two additional benchmarks for testing: **NUSWIDE** (Chua et al. 2009) and **CCV** (Jiang et al. 2011). Note that our train/test split is more challenging than traditional split which is done on the same dataset. NUSWIDE and CCV respectively contain 269,648 images and 9,317 videos across 81 and 20 semantic classes. Note that the number of data in each class varies from tens to thousands. For fair comparison, queries were uniformly sampled from each class. This gave rise to 4,860 and 3,560 queries for NUSWIDE and CCV, respectively. For each dataset, database was considered as the whole dataset except the query. We sampled 10 query sets for averaging the results. We used the advanced DeCAF deep learning visual features for images (Donahue et al. 2013) and the feature dimension was $d = 4,096$. In particular, for each video, we sampled 1 frame image in every 5 frames and the feature vector was the mean of all the sampled frames.

Search Protocol and Metric

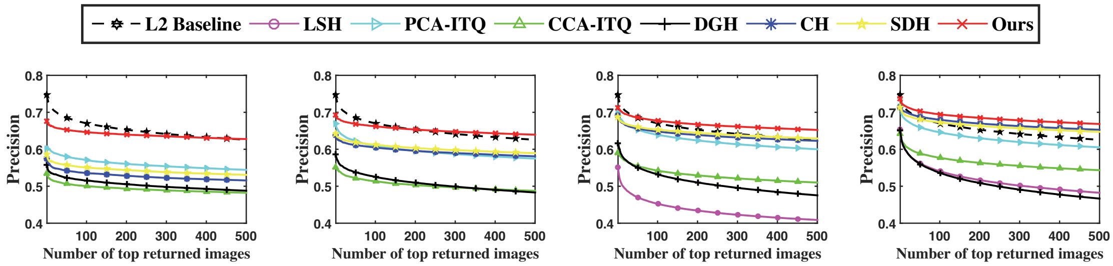
We adopted **Hamming ranking** as our search protocol. All the data in the database are ranked according to their Hamming distance from the query and the top ranked data are returned as the results. Another widely used protocol is Hash lookup, where a lookup table is constructed and results are data within a Hamming radius (*e.g.*, 2). Although Hamming ranking requires linear search time as compared to the constant time in Hash lookup, the former provides better quality measurement of the learned Hamming embedding while the latter only focuses on search speed and fails to handle the case that the bit size is larger than 32 (Wang, Kumar, and Chang 2012). Since our testing data are labeled, for evaluation metric, we used the popular **Precision@K** (K from 1 to 500) with class labels as the groundtruth (Gong et al. 2013).

Compared Methods

We compared the proposed hashing method: **Ours**, against 6 state-of-the-art hashing methods:
LSH (Gionis et al. 1999): Locality Sensitive Hashing. This method models hashing function $\mathbf{W}$ as a Gaussian random matrix.
PCA-ITQ (Gong et al. 2013): this method uses $\mathbf{W} = \mathbf{W}_{\text{pca}}\mathbf{R}$, where $\mathbf{W}_{\text{pca}}$ is the PCA projection and $\mathbf{R}$ is a rotation matrix learned by their proposed iterative quantization.
CCA-ITQ (Gong et al. 2013): this method replaces the PCA projection to CCA projection $\mathbf{W}_{\text{cca}}$, which is learned from two data modalities: the image feature $\mathbf{X}$ and the label $\mathbf{A}$.
DGH (Liu et al. 2014a): the recently proposed Discrete Graph Hashing. This method can be considered as an advanced version of Spectral Hashing (Weiss, Torralba, and Fergus 2009) since it supports large-scale training data and discrete minimization for quantization loss.
CH (Liu et al. 2014b): Collaborative Hashing. It applies collaborative filtering for training hash functions. In particular, we modified this method by applying the collaborative supervision used in this paper. Note that the original CH neglects specially modeling the unobserved labels.
SDH (Shen et al. 2015): Supervised Discrete Hashing. It considers data hashing codes as the features for multi-label supervised training, through which the hashing functions can be learned simultaneously.
Among the above methods, LSH, PCA-ITQ and DGH are unsupervised; CCA-ITQ, CH and SDH are supervised. Although there are many other supervised hashing methods, they were not compared since they cannot be easily extended to multi-label training data. Except LSH and CH, the rest of them have discrete optimization. We implemented these methods using the codes provided by the authors with default parameters. We used **L2-Baseline** which performs search using ℓ_2-norm Euclidean distance between original features as the baseline.

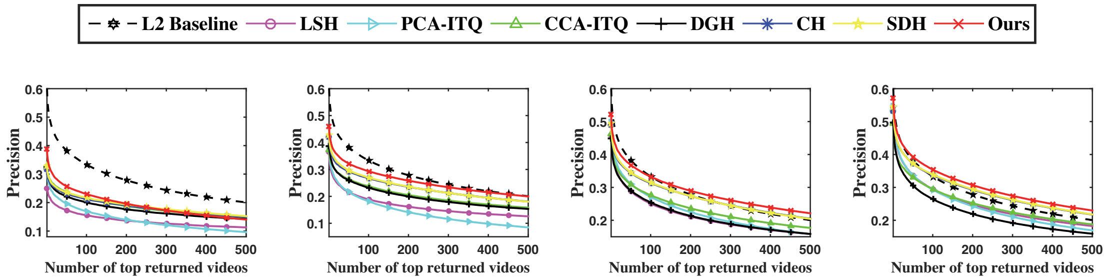
Parameter Sensitivity

We empirically set λ_2 to $1e - 4$ since we did not find any performance drop around $1e - 4$. As compared to λ_2, λ_1 is a more crucial trade-off parameter since it balances between the supervised loss and the quantization loss. As in Figure 5,



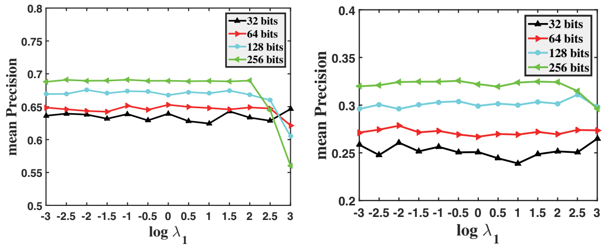
(a) 32 bits (b) 64 bits (c) 128 bits (d) 256 bits

Figure 3: Performance (Precision@K) of various methods with different bit sizes on NUSWIDE.



(a) 32 bits (b) 64 bits (c) 128 bits (d) 256 bits

Figure 4: Performance (Precision@K) of various methods with different bit sizes on CCV.



(a) NUSWIDE (b) CCV

Figure 5: Mean Precision from top 1 to top 500 retrieval results on the two datasets with various bit size and λ_1.

we plot the mean precision, averaged from top 1 to 500, of various bit size and λ_1. We can find that our algorithm is not very sensitive to λ_1. Therefore, in all experiments, for a certain bit size, we set λ_1 to the value with best performance. For the learning rate η used in U-subproblem, we initially set it to $1e - 3$ and used a dynamic learning rate updating heuristic (Qian 1999).

Results

Figure 3 and Figure 4 illustrate the retrieval results of various methods using different bit sizes on both datasets. We can see that our proposed method considerably outperform the other methods, especially the recently proposed CH and SDH. This demonstrates the effectiveness of our two main proposals: collaborative supervision for weak labels and discrete hashing.

First, on both datasets and all bit sizes, we can see that the state-of-the-art supervised methods are generally better than the unsupervised methods. In particular, our method outperforms the recently proposed and competitive supervised method: SDH. Although we both approach discrete optimization, SDH adopts the traditional supervision, which is confused by the unobserved weak labels. This may also explain why CCA-ITQ performs the worst among supervised methods since it directly attempts to model the explicit correlations between image features and label vectors.

Second, as compared to CH, the reason why our method performs much better is discrete optimization. Note that our implementation for CH is the same as the proposed method but without discrete optimization. This demonstrates the effectiveness of minimizing quantization loss in hashing.

Third, we can see that with longer bit size, the supervised hashing performance can even outperform L2 baseline. That is to say, through supervised training, we can perform more accurate retrieval with much lower time and storage cost. This demonstrates the effectiveness of supervised hashing using large training data. Note that the testing datasets NUSWIDE and CCV differ from the training set: SBU, therefore, our promising results on this challenging task offers large potentials in practical hashing applications.

Conclusions

In this paper, we explored a novel hashing framework that uses discrete optimization for learning large and weakly-labeled data. We argued that this is an essential but challenging task since although large training data is beneficial for supervised hashing , it rarely works because it fails to resolve the noisy and sparse nature of weak supervision. We tackled this challenge by adopting the key idea of how collaborative hashing can successfully handle weak relationships. By addressing this motivation, we developed a formulation which jointly models the collaborative supervision and quantization loss. In particular, we proposed an efficient algorithm that explicitly uses discrete optimization to avoid unnecessary quantization loss during optimization. Through training on one million weakly annotated images and testing on two challenging benchmarks, our method considerably outperformed various state-of-the-art hashing techniques.

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